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Relative scales of the GUT and twin sectors in an F-theory model

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Abstract: In this note we analyze the relative scales for the GUT and twin sectors in the F-theory model discussed in ref. [1]. There are a number of volume moduli in the model. The volume of the GUT surface in the visible sector (1) (with the Wilson line GUT breaking) defines the GUT scale $M_G \sim 2 \times 10^{16}$ GeV as the unification scale with precise gauge coupling unification of $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$. We choose the GUT coupling constant, $\alpha_G^{-1} \sim 24$. We are then free to choose the ratio $\alpha_G(2)/\alpha_G(1) = m_1/m_2$ with m_1 and m_2 independent volume moduli associated with the directions perpendicular to the two asymptotic GUT surfaces. We then analyze the effective field theory of the twin sector (2), which may lead to a SUSY breaking gaugino condensate. Of course, all these results are subject to the self-consistent stabilization of the moduli.

Keywords: Differential and Algebraic Geometry, F-Theory, Supersymmetric Effective Theories, Supersymmetry and Duality

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1 Relative scales in F-theory GUT

The effective low energy field theory of an F-theory GUT is defined on a real 10-dimensional manifold $\mathbb{M}^{10} = \mathbb{R}^{3,1} \times B_3$ where B_3 is a smooth complex projective complex 3-fold with ample anti-canonical bundle, that is, a Fano threefold. Gravity fills all of this real 10-dimensional space-time while the GUT theory resides on a smooth two-dimensional anti-canonical complex surface $S_{\mathrm{GUT}} \subseteq B_3$. The GUT surface S_{GUT} is defined by the vanishing of $z \in H^0(K_{B_3}^{-1})$.

1.1 Semi-stable degeneration of the F-theory model

More precisely, as in [1] we consider the family

$$B_{3,\delta} = \mathbb{P}_{[u_0,v_0]} \times B_2.$$

where $B_3 = B_{3,1}$. For the affine coordinates

$$(a,b) \in \mathbb{C}^2 \subseteq \mathbb{P}^1_a \times \mathbb{P}^1_b$$

we set

$$a = \delta^{1/2}\frac{u_0 - v_0}{u_0 + v_0}$$
$$b = \delta^{1/2}\frac{u_0 + v_0}{u_0 - v_0}$$

and consider the closure $\mathbb{P}_{[\delta]}$ of the subset

$$a \cdot b = \delta \in [0,1]$$

so that

$$B_{3,\delta} = \mathbb{P}_{[\delta]} \times B_2.$$

Here B_2 is constructed from the 'last' toric del Pezzo D_6 endowed with its canonical toric metric $g_{(6)}$ as in (1.3) of [2] with respect to which the $\mathbb{Z}_4$-action in [1] is isometric with finite fixpoint set.[1] We then blow up a single general $\mathbb{Z}_4$-orbit, the resulting del Pezzo

$$D_2 \subseteq D_6 \times \mathbb{P}^1$$

where $\mathbb{Z}_4$ acts trivially on $\mathbb{P}^1$ and the metric $g_{(2)}$ on $D_2 = B_2$ is induced by the above inclusion. Thus we have one real degree of freedom in the choice of the scaling constant on the standard SU (2)-invariant metric on $\mathbb{P}^1$.

Similarly $\mathbb{P}^1_a$ carries the standard SU (2)-invariant metric g_a with volume m_1 and $\mathbb{P}^1_b$ carries the standard SU (2)-invariant metric g_b with volume m_2. The metric g_δ on $B_{3,\delta}$ is the one induced by restriction of the metric

$$g_a \oplus g_b \oplus g_{(2)}$$

on $\mathbb{P}^1_a \times \mathbb{P}^1_b \times B_2$. This allows two additional scaling constants, the first giving volume m_1 to the standard SU (2)-invariant metric on $\mathbb{P}_a$ and the second giving volume m_2 to the standard SU (2)-invariant metric on $\mathbb{P}_b$.

The Einstein-Hilbert action is given by

$$S_{\mathrm{EH}} \sim M_*^8 \int_{\mathbb{R}^{3,1}\times B_3} R\sqrt{-g_\delta}\,d^{10}x. \tag{1.1}$$

As a consequence, the four-dimensional Planck constant is given by

$$M_{\mathrm{Pl}}^2 \simeq M_*^8 \cdot Vol\left(B_{3,\delta}\right). \tag{1.2}$$

The semi-stable limit of the F-theory geometry as δ goes to zero is the union of two components or 'gauge sectors'

$$B_3^{(1)} \cup B_3^{(2)}$$

crossing along a copy of B_2 over $(a, b) = (0, 0)$ where

$$\begin{aligned} B_3^{(1)} &= \mathbb{P}^1_a \times B_2 \\ B_3^{(2)} &= \mathbb{P}^1_b \times B_2. \end{aligned}$$

We call $B_3^{(1)}$ with induced metric g_1 the 'visible sector' and $B_3^{(2)}$ with induced metric g_2 the 'hidden or twin sector.' Thus

$$Vol\left(B_{3,0}\right) = Vol\left(B_2\right) \cdot \left(m_1 + m_2\right).$$

1.2 Asymptotic position of S_{GUT}

As δ varies, the GUT surface $S_{\mathrm{GUT},\delta}$ is defined by the vanishing of

$$z_\delta = \delta \cdot z + (1-\delta) \cdot \left(u_0^2 - v_0^2\right) \cdot q \in H^0\left(K_{B_{3,\delta}}^{-1}\right)$$

[1] This metric is not the Kähler-Einstein metric on D_6. See [3].

where $q \in H^0(K_{B_2}^{-1})$ is a section in the (-1)-eigenspace for the $\mathbb{Z}_4$-action on B_2. We let $C \subseteq B_2$ denote the smooth genus-one curve defined by the vanishing of q.

Here the gauge action is given by

$$S_{\text{gauge}} \sim -M_*^6 \int_{\mathbb{R}^{3,1}\times B_{3,0}} \left(\mathrm{Tr}(F_1^2)\sqrt{-g_1} + \mathrm{Tr}(F_2^2)\sqrt{-g_2}\right) \delta^2(z_0)\; d^{10}x.$$

F_i denotes the (limiting) curvature tensor of the Yang-Mills connection on the i-th gauge sector $B_3^{(i)}$. Also $S_{\text{GUT},0} \supseteq S_1 \cup S_2$ where

$$\begin{aligned} S_1 &:= (\{a=\infty\}\times B_2) \cup \left(\mathbb{P}_a^1 \times C\right) \\ S_2 &:= (\{b=\infty\}\times B_2) \cup \left(\mathbb{P}_b^1 \times C\right). \end{aligned}$$

Therefore

$$\mathrm{Vol}\,(S_i) = M_G\,(i)^{-4} = \mathrm{Vol}\,(B_2) + m_i \cdot \mathrm{Vol}\,(C)\,.$$

(See appendix A for more detail on the relationship between $S_1 \cup S_2$ and $S_{\text{GUT},0}$.)

2 Scaling the effective 4-D theory

Hence in the effective 4-dimensional theory we should have GUT coupling constant

$$\alpha_G(i)^{-1} \sim M_*^6\ \mathrm{Vol}(S_i)\ R_{\perp i}^2, \quad i=1,2$$

where $R_{\perp i}^2 \equiv m_i$ is the size of the perpendicular scale for the gauge sectors 1, 2. This calculation is complicated by the fact that each S_i has two components whose perpendicular scales do not coincide geometrically nor numerically. The perpendicular scale to the component $S_i \subseteq B_3^{(i)}$ that projects onto B_2 is clearly m_i. However the perpendicular scale for the $\mathbb{P}^1 \times C \subseteq B_i$ must be defined. We do that by recalling that the defining section q of the curve $C \subseteq B_2$ is a section of the anti-canonical bundle of B_2 and this bundle has a metric induced from the fixed toric metric on B_2. Thus we define the perpendicular scale for the $\mathbb{P}^1 \times C$ component to be norm

$$\int_{B_2} |q|^2\,.$$

Substituting we obtain

$$\alpha_G(i)^{-1} \sim M_*^6 \left(\mathrm{Vol}(B_2)\cdot m_i \ + \mathrm{Vol}\,(C)\cdot m_i \cdot \int_{B_2} |q|^2\right), \quad i=1,2$$

We then find

$$\begin{aligned} \alpha_G\,(i)\,M_{\mathrm{Pl}}^2 \sim & M_*^2 \frac{\mathrm{Vol}\,(B_2)\,(m_1+m_2)}{\mathrm{Vol}(B_2)\cdot m_i + \mathrm{Vol}\,(C)\cdot m_i \cdot \int_{B_2} |q|^2} \\ =& M_*^2 \left(\frac{m_1+m_2}{m_i}\right) \frac{\mathrm{Vol}\,(B_2)}{\mathrm{Vol}(B_2) + \mathrm{Vol}\,(C)\cdot \int_{B_2} |q|^2}. \end{aligned}$$

Therefore the relative size of the GUT coupling constants and the GUT scales for the visible and twin sectors depends on the relative sizes of the perpendicular directions in B_3 to the GUT surface.

Let's define the sector labeled (1) as the visible sector with GUT coupling constant, $\alpha_G(1)^{-1} = 24$ at the GUT scale $M_G(1) = 2 \times 10^{16}\ GeV$. Then the twin sector is sector (2). The ratio $\alpha_G(2)/\alpha_G(1) = m_1/m_2$ with $M_G(2) > M_G(1)$. Let's take $M_G(2) = 3 \times 10^{16}\ GeV$ and $\alpha_G(2)^{-1} = 8.7$ or $m_1/m_2 = 2.8$. Below the scale $M_G(2)$ the effective field theory is $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)_Y$, just as in the visible sector. However the twin QCD coupling will become strong at a scale much greater than the visible QCD scale. The effective twin QCD theory has $N_C = 3$ and $N_{flavors} = 6$. Hence it is described by Seiberg duality [6]. In the magnetic phase, we have an effective superpotential given by

$$W = q^{ia} T_{i,a}{}^{j,b} \bar{q}_{j,b} + \lambda^u_{ij} q^{ia} H_{ua} \bar{q}_{j1} + \lambda^d_{ij} q^{ia} H_{da} \bar{q}_{j2} \tag{2.1}$$

where q ($\bar{q}$) are left-handed color triplets (anti-triplets) with the family index, $i = 1, 2, 3$, and $\mathrm{SU}(2)_{isospin}$ index, $a = 1, 2$. When $\langle q \rangle_0 = \langle \bar{q} \rangle_0 = 0$, the theory has a flat direction for the fundamental meson field, $T_{i,a}{}^{j,b}$. Note, since the twin electroweak group is gauged, we should identify $T_{i,a}{}^{j,1} \equiv (T_{ua})_i{}^j$ and $T_{i,a}{}^{j,2} \equiv (T_{da})_i{}^j$. The twin supersymmetric $\mathrm{SU}(2) \times \mathrm{U}(1)_Y$ gauge interactions introduce a quartic potential for T_u, T_d, H_u, H_d such that there is a flat direction for $\langle (T_{u1})_i{}^j \rangle = \langle (T_{d2})_i{}^j \rangle = T \delta_i{}^j$ and $H_{u1} = H_{d2} = T$. Then all twin quarks and leptons obtain mass at the scale T and, moreover, the twin electroweak gauge symmetry is broken down to twin $\mathrm{U}(1)_{EM}$. For $T \sim M_G(2)$, we find a twin gluino condensate occurs at the scale $\Lambda_{\mathrm{tQCD}} = T \exp(-\frac{2\pi}{9\alpha_G(2)}) \sim 6.9 \times 10^{13}\ GeV$.

We expect that the effective 4D QCD Lagrangian contains a term

$$L \supset \frac{1}{2} \int d^2\theta \sum_{i=1}^{2} \left(\frac{S(i)}{4} \mathrm{Tr} W^\alpha W_\alpha(i) + \text{h.c.} \right) \tag{2.2}$$

with

$$S(i) = \frac{1}{4\pi\alpha_G(i)} + i\theta = e^{\ln(K(i) m_i) - \phi} + ib, \tag{2.3}$$

where ϕ, b is the dilaton, axion fields, and m_i is as above the volume of the $\mathbb{P}^1$ direction perpendicular to the GUT surface in the visible and twin sectors and $K(i) \sim M_*^6\ M_G(i)^{-4}$. We also expect a non-perturbative superpotential term of the form [7–9]

$$W_{\mathrm{NP}} \supset A[e^{-8\pi^2 S(2)/9} T]^3. \tag{2.4}$$

As a consequence, the twin QCD condensate will contribute SUSY breaking effects to both the twin and visible sectors of the theory. In this local SUSY theory, we find an effective low energy SUSY breaking scale given by

$$m_{3/2} = \Lambda^3_{\mathrm{tQCD}}/M^2_{\mathrm{Pl}} \sim 60\ TeV. \tag{2.5}$$

Of course, whether supersymmetry is broken (or not) depends on stabilizing all the moduli.

The low energy supersymmetric theory contains 3 families of twin neutrino superfields (assuming that the three right-handed neutrinos obtain mass near the GUT scale),

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19 chiral charged and neutral Higgs superfields (which include the massless components of H_u, H_d and T_u, T_d), and the twin photon superfield. Renormalizing from $M_G(2)$ we find $\alpha^{tEM}(m_{3/2}) \sim 1/105$. There do not appear to be any portals to the twin sector. Clearly, the twin sector introduces new candidates for dark matter, but a complete analysis of the cosmological implications of this sector for the theory is beyond the scope of the present paper.

3 Conclusion

In conclusion, we have analyzed the relative scales of the visible and twin sectors in the global F-theory GUT with Wilson line breaking given in [1]. We have found that there is sufficient freedom to have independent GUT scales and couplings in order to have interesting physics coming from the twin sector. In particular, if we assume that the GUT coupling for the twin sector is larger than that of the visible sector, then it is possible to spontaneously break the twin electroweak theory at the GUT scale with all twin quarks and charged leptons obtaining mass at that scale. In addition, a twin gluino condensate can then occur at a scale of order $\Lambda_{\mathrm{tQCD}} \sim 6.9 \times 10^{13}\, GeV$ which leads to an effective low energy SUSY breaking scale, $m_{3/2} = \Lambda_{\mathrm{tQCD}}^3 / M_{\mathrm{Pl}}^2 \sim 60\, TeV$ which affects both the twin and visible sectors. There is clearly more analysis that needs to be done on the consequences of these results, including the stabilization of moduli, which we leave for the future. For example, the model also includes 11 D_3 branes and fluxes which must be considered [10].

A Asymptotic position of S_{GUT} with regards to the Heterotic theory

The Heterotic dual is the smooth Calabi-Yau threefold $[V_3 = B_3^{(1)} \cap B_3^{(2)}]$ over $a = b = 0$. Each $B_3^{(i)}$ encodes the structure of an E_8-bundle with Yang-Mills connection on the Heterotic model V_3 via the equivalence of Yang-Mills E_8-bundles and dP_9-bundles given by the dictionary in §4.2 of [4]. A slightly subtle point in the encoding is that the factor q in z_0 is irrelevant to the determination of the E_8-bundle, since over each point $b_2 \in B_2$, $q(b_2)$ simply re-scales the vector $[s, t]$ in the Weierstrass form

$$\left[y^2 = 4x^3 - \left(g_2 t^4 - \beta_1 s t^3 - \ldots - \beta_4 s^4\right) x - \left(g_3 t^6 - \alpha_2 s^2 t^4 - \ldots - \alpha_6 s^6\right)\right]$$

for the equation of each dP_9-fiber of $B_3^{(i)}/B_2$. Thus the Weierstrass form, and so the isomorphism class of the dP_9-bundle, is unaffected. Said otherwise, Heterotic "S_{GUT}" is simply the union of the section $\{a = \infty\}$ of $B_3^{(1)}/B_2$ and the section $\{b = \infty\}$ of $B_3^{(2)}/B_2$.

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